Logistics:

Mid Sem: 25
End Sem: 50

CWS (25):

Attendance: 5 → >95% → 5
90 − 95 → 4
85 − 90 → 3
80 − 85 → 2
75 → 80 → 1
<75 → 0

Assignment: 10
Test: 10

Webpage: Notes, Syllabus

Theory of Computation:

Symbol — a, b, c, d, 0, 1, 2 ..... — Basic Building Block (letters / numbers)

Alphabet — 0, 1 — Σ — Subset of Symbols

String — $\Sigma = \{a, b, c\}$ — a, ab, ac, bc, acc, abc, ... — Sequence of Alphabets

How many strings are possible of length n with {a, b} alphabets?

length: 3

$\underset{\substack{a,b \\ 2}}{\_\_} \; \underset{\substack{a,b \\ 2}}{\_\_} \; \underset{\substack{a,b \\ 2}}{\_\_} = 2^3$

length: n

$\underbrace{\underset{2}{\uparrow}\ \underset{2}{\uparrow}\ \underset{2}{\uparrow}\ \_\ \_\ \_\ \_\ \_\ \cdots\ \_}_{n \text{ terms}}$

2^n

$\Sigma = \{a, b\}$

$|\Sigma|$ = no of alphabets

no. of strings of length n : $|\Sigma|^n$

<u>Language:</u> Collection of Strings

L_1 : Set of all strings of length 2

$\Sigma = \{a, b\}$

Strings: a, b, aa, baa, baba, ...

$L_1 = \{aa, ab, ba, bb\}$ → finite languages

L_2 : Set of all strings of length 3

$\Sigma = \{a, b\}$

$L_2 = \{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

L_3 : Set of all strings which are starting with a

$\Sigma = \{a, b\}$

$L_3 = \{a, aa, ab, abb, aba, abbb \ldots\ldots\}$ → Infinite language

Powers of Σ

$\Sigma = \{a, b\}$

Σ^1 = set of all strings over Σ of length 1

$= \{a, b\}$

Σ^2 = set of all strings over Σ of length 2

$= \{aa, ab, ba, bb\}$

$\Sigma^3 = \Sigma . \Sigma . \Sigma$

$= \{a, b\} \{a, b\} \{a, b\}$

$= \{aaa, aab, aba, abb \ldots \ldots \}$

$|\Sigma^3|$ = Cardinality of Σ^3 = 8

Σ^n = n length strings

Σ^0 = Set of all strings of length 0

$= \{\varepsilon\}$

epsilon is a spcl symbol of length 0

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \ldots \ldots \Sigma^n$

$= \{\varepsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \cup \ldots \ldots$

Set of all strings possible over $\{a, b\}$ of all length.

Case 1 : finite

L = 2 length strings

$\{aa, ab, ba, bb\}$

'bc' ? ✗

Case 2 : Infinite

$L_3 = \{a, ab, abb, aba, \ldots\}$

'bc'

Does string belong to the language or not ?

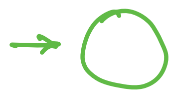
Machine

Given a language L, you need a finite representation which can be stored in memory and by using it you should be able to tell if string is present in language or not.

Finite Automata

State: 

Final State 

 Initial State

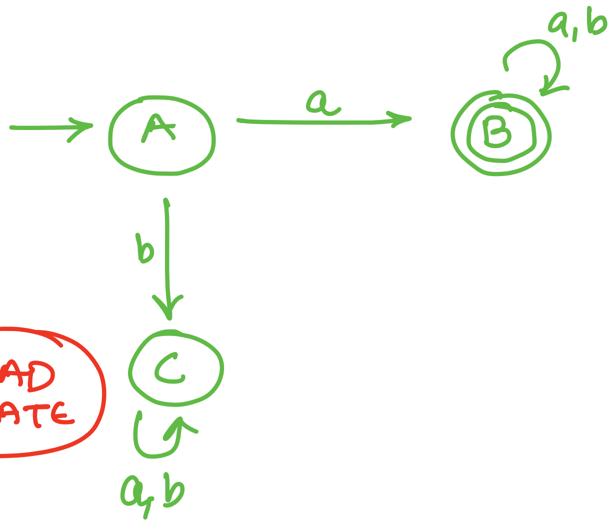
FA:

L = Strings starting with 'a'

$\Sigma = \{a, b\}$

'abba'

'baa'



On every state & on every input alphabet : transition present

DEAD STATE

from where we can never reach final state

abba : $A \xrightarrow{a} B \xrightarrow{b} B \xrightarrow{b} B \xrightarrow{a} B \rightarrow$ final State

Accepted

baa : $A \xrightarrow{b} C \xrightarrow{a} C \xrightarrow{a} C \rightarrow$ non final State

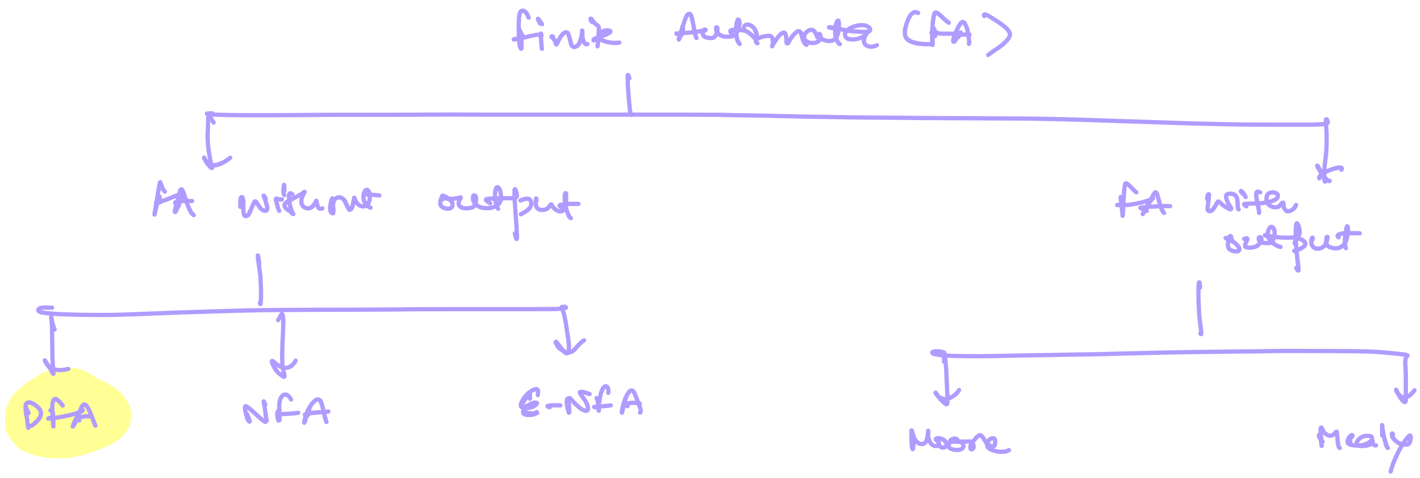
Not accepted

$\Downarrow$

Not in the language

All the strings which are present in the language should be accepted

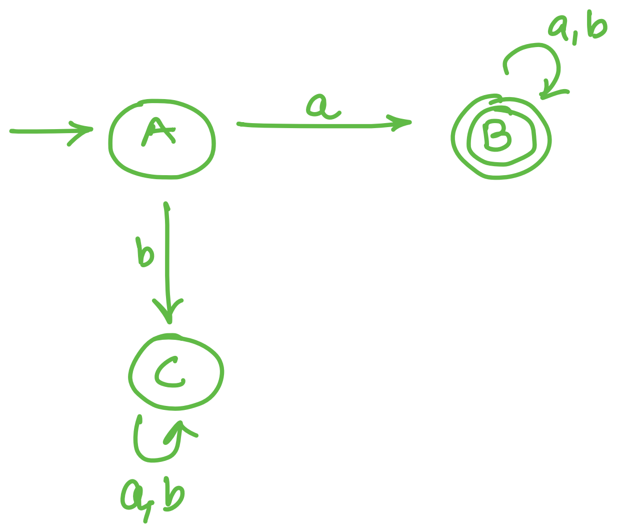
All the strings which are not present in the language should not be accepted.



Deterministic finite Automata :

$\rightarrow$ FA which contains $Q, \Sigma, \delta, q_0, F$

- Q: finite set of all states
- Σ: input alphabet
- δ: transition fx^n
- q_0: initial state
- F: final state



$Q = \{A, B, C\}$

$\Sigma = \{a, b\}$

$q_0 = \{A\}$

$F = \{B\}$

$\delta : Q \times \Sigma \longrightarrow Q$

$\{A, B, C\} \times \{a, b\}$

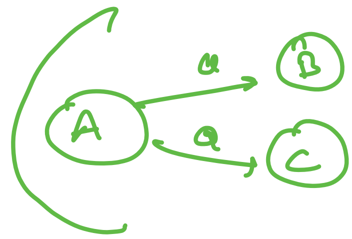
$A, a \longrightarrow B$
$A, b \longrightarrow C$
$B, a \longrightarrow B$
$B, b \longrightarrow B$
$C, a \longrightarrow C$
$C, b \longrightarrow C$

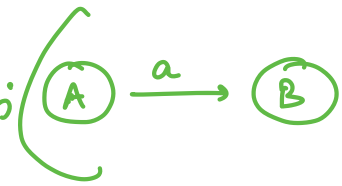
Type 1

Q: DFA which accepts set of all strings over $\{a, b\}$ of length 2.

NFA
1, 0, many

exactly 1 transition

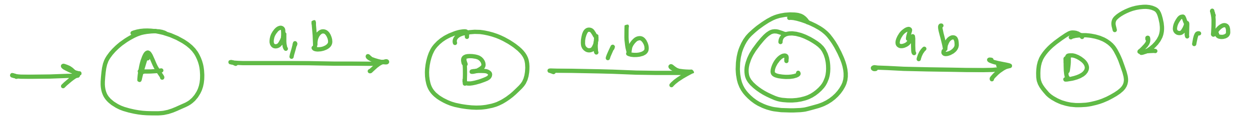




'ba' $\longrightarrow$ $A \xrightarrow{b} B \xrightarrow{a} C$ → final state accepted → present in the language

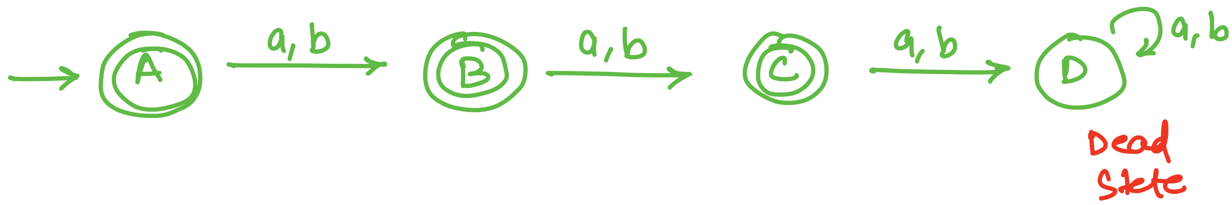
'baba' $A \xrightarrow{b} B \xrightarrow{a} C \xrightarrow{b} D \xrightarrow{a} D$ → non final state not accepted → not present in language.

Q: String length atleast 2.
$\Sigma = \{a, b\}$



Q: String length atmost 2

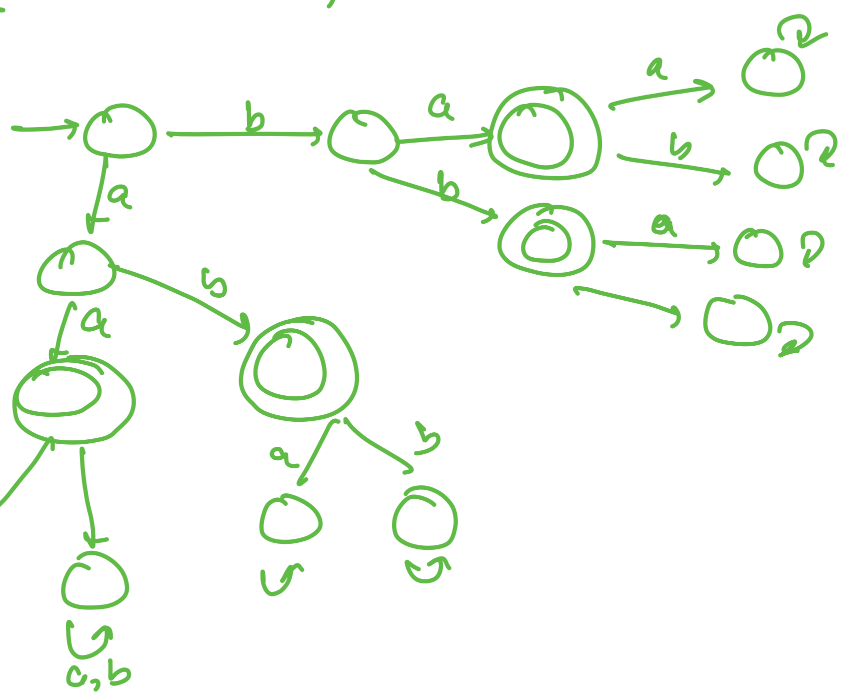
$\Sigma = \{a, b\}$



ε: accept

initial state ↷ final state

Exactly 2: aa, ab, ba, bb



multiple DFAs possible

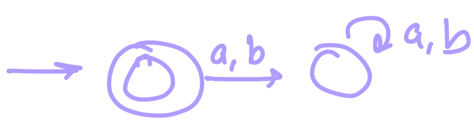
minimal DFA : 1.

length exactly n : min $n+2$ states

atleast n : min $n+1$ states

atmost n : min $n+2$ states

$n = 0$



Type 2

Q: DFA that accepts set of all strings over $\{a, b\}$

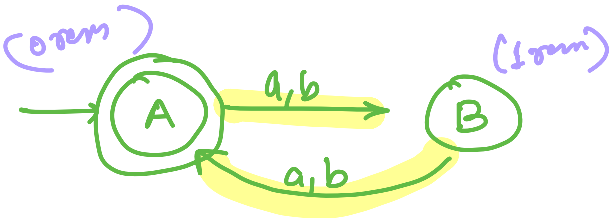
such that length of String mod 2 = 0 .

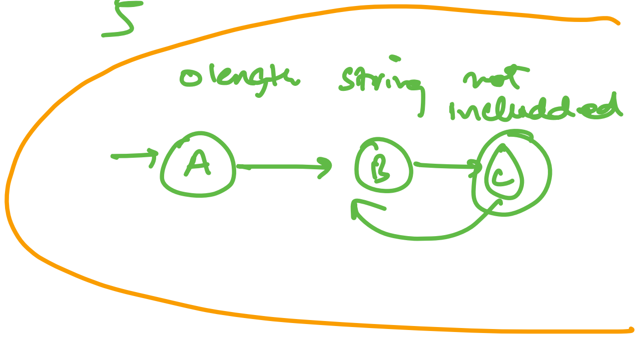
$\ell \% 2 = 0$

↳ Even length

0, 2, 4, 6, 8 . . . . .

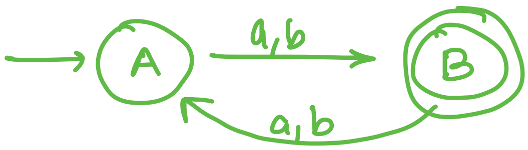
{ε, aa, ab, ba, bb, aaaa, aaab . . . . }





Q: String length mod 2 = 1

Odd length

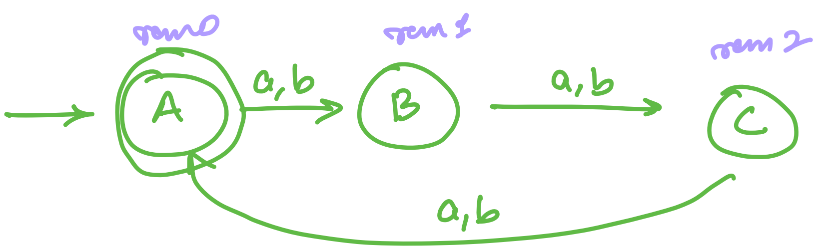


Q: String length mod 3 = 0

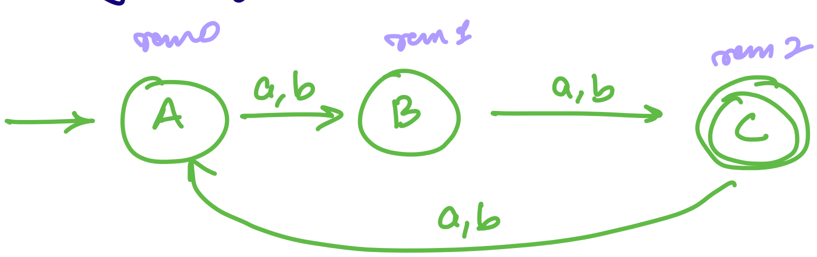
OR

Divisible by 3

% 3 → 0
→ 1
→ 2



Q: String length mod 3 = 2

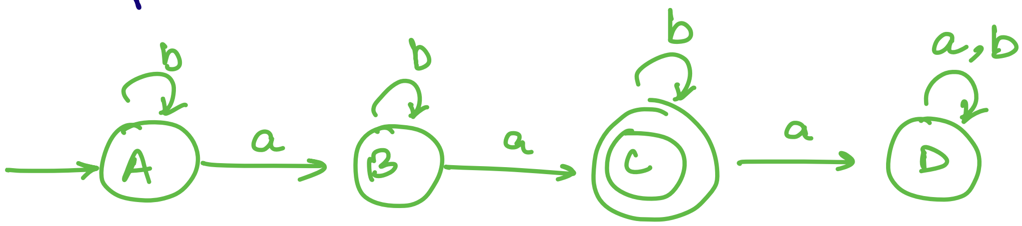


|w| mod n = 0 minimal no of states = n

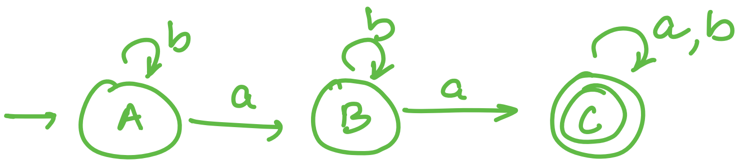
Type 3

$\Sigma = \{a, b\}$

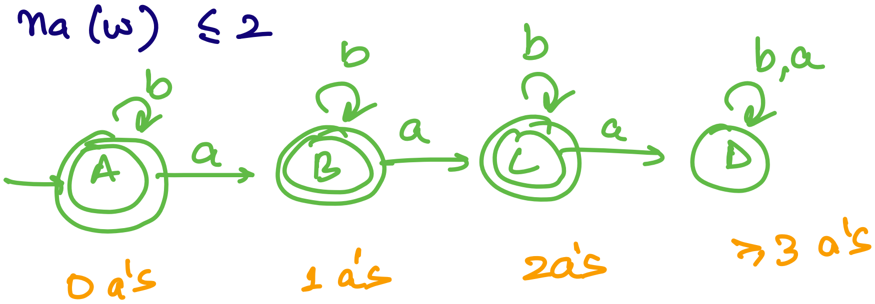
Q: no. of a's are 2 $\quad n_a(\omega) = 2$



Q: $n_a(\omega) \geq 2$



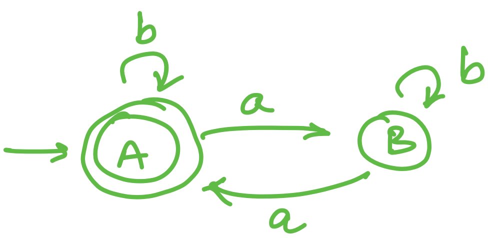
Q: $n_a(\omega) \leq 2$



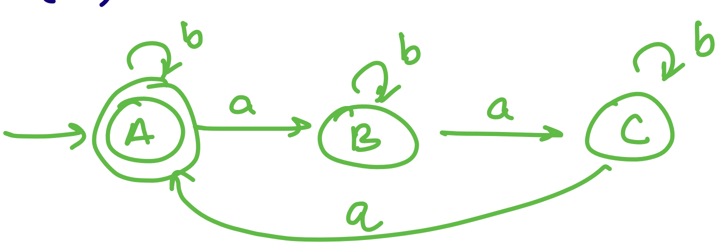
Type 4

Q: $n_a(\omega) \bmod 2 = 0$

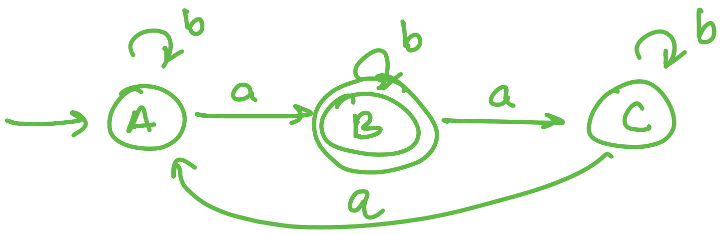
count of no. of a's should be even.



Q: $n_a(\omega) \bmod 3 = 0$



Q: $n_a(w) \bmod 3 = 1$



Q: $n_a(w) \cong 0 \bmod 2 \longrightarrow n_a(w) \bmod 2 = 0 \longrightarrow n_a$ even

and

$n_b(w) \cong 0 \bmod 2 \longrightarrow n_b(w) \bmod 2 = 0 \longrightarrow n_b$ even